

## The Mediated Character of Immediate Inferences

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**Abstract:** In the present article we aim to illustrate the mediated character of inferences designated as being immediate. We analyze immediate inferences through equivalence, respectively deducted immediate inferences based on the oppositional hexagon relations. For both types of inferences, the illustration is made using both Aristotelian and composite sentences. The result of this approach indicates that, in the cases of both of the two types of immediate inferences being discussed, it comes as a mediated inference, namely a two-premise inference, following the hypothetic-categorical inference model and the disjunctive-categorical model from the logic of composite sentences. In conclusion, the most frequently used immediate inferences are built on three hypothetical dyadic relations and three disjunctive relationships, and the composite sentence (which expresses the relationship) is the absent, but tacitly assumed, premise. We refer to premise without which the conclusion could not be established. The absence of the premise that expresses the logical relation between the sentences involved in the inference ensures the immediate aspect of the approach. So, in reality, immediate inferences are incomplete, mediated approaches.

**Keywords:** immediate inference, mediated, equivalence, opposition

Deductive inferences with a single premise are considered *immediate*, because it is assumed that the sentence taken as a conclusion is justified, without the intermediation provided by other information, other premises. In the case of immediate inferences, knowing the truth value of a given sentence (the premise), one can deduce the truth value of another sentence (the conclusion) with which it is in a certain logical relation. If the definition wouldn't have had the last specification, the inference would've been worth the epithet of immediate. In reality, the logical relation - on which the derivation is made - announces the absent premise, a composite sentence which plays the role of a law / a rule, on which truth values are conjugated.

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The inference of the  $R a E \rightarrow \sim(R e E)$  type is considered immediate, for having only one premise. However, it is recognized as an inference by opposition, that is, on the basis of the square / hexagonal relations, more precisely, it is an immediate inference by contrariety! Where is the justifying relation? It is tacitly assumed. In reality, the falsity of the conclusion of the given inference is founded on the contrariety relation between the two universal sentences and the truth of the other components of the relation:  $[(R a E \uparrow R e E) \& R a E] \rightarrow \sim(R e E)$ . Since  $a$  and  $e$  sentences are contrary, and  $a$  is true, it follows that the sentence  $e$  is false.

Because the sentences *All Romanians are Europeans* and *No Romanians are Europeans* cannot be both true at the same time, and the sentence *All Romanians are Europeans* is true, it follows that the sentence *No Romanians are Europeans* is false. With what does, this scheme, differ from the *mediated* deductive inferences of the disjunctive-categorical type:  $\{[(p \uparrow q) \& p] \rightarrow \sim q\}$ ? With nothing, but it is put into the circuit without the first premise (which contains the rule of conjugation of truth values), though it uses it. Without the first premise, the sequence would not be possible.

The knowledge of these types of inferences is necessary to any specialist, but especially for educators and jurists. The former train the students in the correct operations of thinking, forming cognitive structures, a style of understanding and use of information, etc. On the other hand, how could a jurist establish the truth, if he did not have the ability to quickly and efficiently understand a text, to understand the formal shortcuts for reasoning and linking information, to identify equivalences, formal and informational, to correctly understand the conditions of the types of implications, the differences between the types of logical oppositions, for identification of an insufficient support of a thesis, etc.? And in this area, as in all others, the solution comes with an adequate mind education (Toma 2017).

## IMMEDIATE INFERENCES THROUGH EQUIVALENCE

In this class are included the inferences built based on the relation of equivalence between two sentences, resulting from operations of conversion, obversion, (partial and total) contraposition, and (partial and total) inversion. The given sentence and the sentence resulting from these operations (the conversed, the obversed, the counterposed, and the inversed) are in a relation of equivalence. As such, the sentence composed on the basis of the equivalence between a given

sentence and the resulting sentence, by one of the operations listed above, represents the tacitly assumed premise, which ensures the mediated character of this type of inferences.

*Immediate inferences through conversion:* Through conversion, from a sentence  $Rxy$ , an *equivalent* sentence  $R'yx$  is constructed, where  $R$  is the relation, and  $x, y$  are *variable terms*, in the case of Aristotelian sentences, they are *propositional variables*, for composite sentences. If the given sentence contains a symmetric relationship, of the type  $e, i$ , or  $p \uparrow q, p \& q, p w q$ , etc. this remains unchanged in the converse, namely  $R x y \equiv R y x$ .

The conversed Aristotelian sentences are:  $R a E \rightarrow E i R$ ;  $R e E \equiv E e R$ ;  $R i E \equiv E i R$ . The sentence  $o$  cannot be converted correctly: for the  $\bar{R} o E$  form, the converse  $E o R^+$  has the term  $R$  extended.

The given sentence and her converse being equivalent, knowing the truth value of one, the truth value of the other can be inferred, the conclusion. So, the immediate inferences through conversion, with Aristotelian sentences, can be:  $R a E \rightarrow E i R$ ;  $R e E \rightarrow E e R$ ;  $R i E \rightarrow E i R$ , respectively  $\sim(R a E) \rightarrow \sim(E i R)$ ;  $\sim(R e E) \rightarrow \sim(E e R)$ ;  $\sim(R i E) \rightarrow \sim(E i R)$ .

If  $R a E$  is true, its converse,  $E i R$ , will be true as well: if it is true that *All Romanians are Europeans*, then it is true that *Some Europeans are Romanian*, namely  $R a E \rightarrow E i R$ , which passes as an immediate inference through conversion. If  $R e E$  is false, its converse,  $E e R$  will be false as well: if it is false that *No Romanians are Europeans*, then it will be false that *No Europeans are Romanians*, namely  $\sim(R e E) \rightarrow \sim(E e R)$ , another inference through conversion.

In reality, as we have said, the above sequences are possible only because we know the logical relation (of equivalence) between the two sentences, one as a premise, and the other as a conclusion. The conclusion of *immediate* inferences through conversion is mediated by the logical equivalence between the given sentence and its converse. Therefore, the inferential schemes above, lack the first premise.

Actually, they look like this:  $R a E \equiv E i R$ ;  $R a E \vdash E i R$ ;  $R e E \equiv E e R$ ;  $\sim(R e E) \vdash \sim(E e R)$ ;  $R i E \equiv E i R$ ;  $R i E \vdash E i R$ , etc.

In the logic of unanalyzed sentences, the things are the same. The conclusion of immediate inferences is justified by the equivalence relation between the given sentence and its converse.

The converses for symmetric sentences preserve the relation of the given sentence:  $(p \vee q) \equiv (q \vee p)$ ;  $(p \equiv q) \equiv (q \equiv p)$ ;  $(p w q) \equiv (q w p)$ ;  $(p \& q) \equiv (q \& q)$ , etc., and for non-symmetrical relations the converse

is calculated using the truth table,  $(p \rightarrow q) \equiv (q \leftarrow p)$ ;  $(p \nrightarrow q) \equiv (q \nleftarrow p)$ , etc. Therefore, immediate inferences of the type  $(p \vee q) \rightarrow (q \vee p)$ ;  $\neg(p \rightarrow q) \rightarrow \neg(q \leftarrow p)$ , etc. are inferences *mediated* by the equivalence relation:  $\{(p \vee q) \equiv (q \vee p)\} \& (p \vee q) \rightarrow (q \vee p)$ ;  $\{(p \rightarrow q) \equiv (q \leftarrow p)\} \& \neg(p \rightarrow q) \rightarrow \neg(q \leftarrow p)$ , etc.

Whether an inference is completely expressed or not, its correctness is given by fulfilling the requirements of the operations by which equivalent sentences are constructed, with a given sentence and by the correct conjugation of the truth values. Given the exceptions to the rule, in the case of some operations, as well as the overall approach, immediate inferences through equivalence, with simple sentences, as well as those with composite sentences can be an important source of errors (Toma 2004, 94-102).

*Immediate inferences through obversion:* Through obversion, from a  $Rxy$  sentence, an *equivalent* sentence  $R'x \sim y$  is constructed. Aristotelian sentences change their quality, having a negative predicate:  $L a A \equiv L e \sim A$ ;  $L e A \equiv L a \sim A$ ;  $L i A \equiv L o \sim A$ ;  $L o A \equiv L i \sim A$ . The corresponding immediate inferences are:  $L a A \rightarrow L e \sim A$ ;  $L e A \rightarrow L a \sim A$ ;  $L i A \rightarrow L o \sim A$ ;  $L o A \rightarrow L i \sim A$ , in the case the given sentence is true. If it is true that *lazy people are unpopular*, then it is true that *lazy people are not sympathetic*, etc.

If the given sentence is false, the corresponding immediate inferences are:  $\neg(L a A) \rightarrow \neg(L e \sim A)$ ;  $\neg(L e A) \rightarrow \neg(L a \sim A)$ ;  $\neg(L i A) \rightarrow \neg(L o \sim A)$ ;  $\neg(L o A) \rightarrow \neg(L i \sim A)$ . As with the previous operation, the inferences through obversion have a *mediated* character, each sequence being possible based on the equivalence relation between the given sentence and its obverse:  $L a A \equiv L e \sim A$ ;  $L a A \vdash L e \sim A$ ;  $L a A \equiv L e \sim A$ ;  $\neg(L a A) \vdash \neg(L e \sim A)$ , etc.

For composite sentences, the obverse is identified using the truth table:  $(p \& q) \equiv (p \nrightarrow \sim q)$ ;  $(p \nrightarrow \sim q) \equiv (p \& q)$ ;  $(p \vee q) \equiv (p \leftarrow \sim q)$ ;  $(p \leftarrow \sim q) \equiv (p \vee q)$  etc. The mediated character of the inferences of the type  $(p \& q) \rightarrow (p \nrightarrow \sim q)$ ;  $\neg(p \vee q) \rightarrow \neg(p \leftarrow \sim q)$  etc. has been highlighted above:  $(p \& q) \equiv (p \nrightarrow \sim q)$ ;  $(p \& q) \vdash (p \nrightarrow \sim q)$ ;  $(p \vee q) \equiv (p \leftarrow \sim q)$ ;  $\neg(p \vee q) \vdash \neg(p \leftarrow \sim q)$ , etc.

*Immediate inferences through total contraposition:* Through total contraposition, from a  $Rxy$  sentence, an *equivalent* sentence of  $R' \sim y \sim x$  is constructed. For Aristotelian sentences, the result is:  $H a S \equiv \sim S a \sim H$ ;  $H e S \equiv \sim S o \sim H$ ;  $H o S \equiv \sim S o \sim H$ , the sentence *i*, cannot be correctly subjected to the operation.

The sentence *the diligent are sympathetic* is logically equivalent with *the unpopular are lazy*. Immediate implications using tacitly the equivalence relation resulting from total contraposition,  $H a S \rightarrow \sim Sa \sim H$ ;  $H o S \rightarrow \sim S o \sim H$ ;  $\sim(H a S) \rightarrow \sim(\sim S a \sim H)$  etc. have, as in the previous cases, a mediated character:  $H a S \equiv \sim S a \sim H$ ;  $H a S \vdash \sim S a \sim H$ ;  $H a S \equiv \sim S a \sim H$ ;  $\sim(H a S) \vdash \sim(\sim S a \sim H)$ , etc.

For a composite sentence, an equivalent sentence can be constructed by total contraposition, using the truth table:  $(p \rightarrow q) \equiv (\sim q \rightarrow \sim p)$ ;  $(p \& q) \equiv (\sim q \downarrow \sim p)$ , etc. The sentence *if it rains, the soil is wet* is logically equivalent to the proposition *if the earth is not wet, it means it has not rained*. The corresponding inferences, used as immediate,  $(p \rightarrow q) \rightarrow (\sim q \rightarrow \sim p)$ ;  $(p \& q) \rightarrow (\sim q \downarrow \sim p)$ ;  $\sim(p \rightarrow q) \rightarrow \sim(\sim q \rightarrow \sim p)$  etc. have a premise tacitly assumed, being mediated:  $\{(p \rightarrow q) \equiv (\sim q \rightarrow \sim p)\} \& (p \rightarrow q) \rightarrow (\sim q \rightarrow \sim p)$ ;  $\{((p \& q) \equiv (\sim q \downarrow \sim p)) \& (p \& q)\} \rightarrow (\sim q \downarrow \sim p)$ ;  $(p \vee q) \equiv (\sim q \uparrow \sim p)$ ;  $\sim(p \vee q) \vdash \sim(\sim q \uparrow \sim p)$ , etc.

*Immediate inferences through total inversion*: From a  $Rxy$  sentence, an equivalent logical sentence  $R' \sim x \sim y$  is constructed by total inversion. Universal sentences become particular sentences of the same quality, but with negative terms:  $H a S \equiv \sim H i \sim S$ ;  $H e S \equiv \sim H o \sim S$ . The sentence *the diligent are sympathetic* is logically equivalent to the proposition that *some lazy people are unpopular*. Particular sentences cannot be fully reversed correctly. Immediate inferences  $H a S \rightarrow \sim H i \sim S$ ;  $H e S \rightarrow \sim H o \sim S$  have a mediated character, and in this case, using two premises to justify the conclusion:  $\{[(H a S \equiv \sim H i \sim S) \& H a S] \rightarrow \sim H i \sim S\}$ ;  $H e S \equiv \sim H o \sim S$ ;  $H e S \vdash \sim H o \sim S$ ;  $H a S \equiv \sim H i \sim S$ ;  $\sim(H a S) \vdash \sim(\sim H i \sim S)$ , etc.

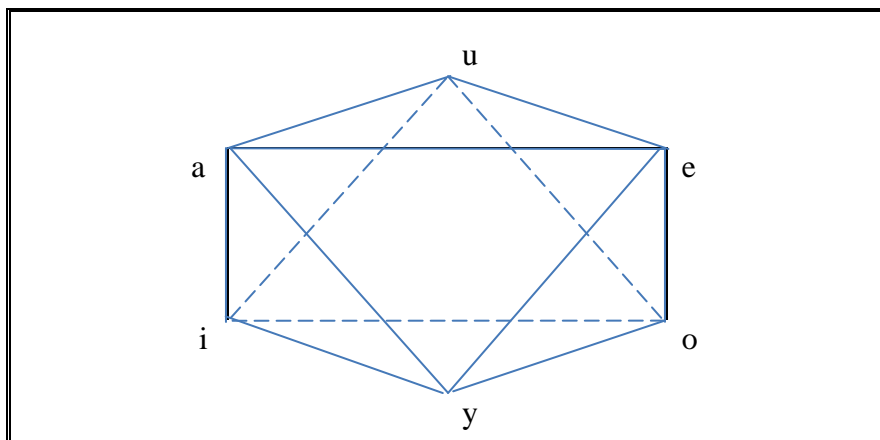
The situation is similar in the case of inferences with compound sentences. The conclusion is the total inverse, and its truth is based on the equivalence relation between the initial sentence and its inverse, as well as on knowledge of the truth value of the initial sentence. The equivalences given by the total inversion  $(p \rightarrow q) \equiv (\sim p \leftarrow \sim q)$ ;  $(p \vee q) \equiv (\sim p \uparrow \sim q)$ ;  $(p \equiv q) \equiv (\sim p \equiv \sim q)$ , etc. will be the absent premises from the immediate inferences:  $(p \rightarrow q) \rightarrow (\sim p \leftarrow \sim q)$ ;  $(p \vee q) \rightarrow (\sim p \uparrow \sim q)$ ;  $\sim(p \vee q) \rightarrow \sim(\sim p \uparrow \sim q)$  etc. *If it rains, the earth is wet, so it did not rain, if the earth is not wet*. By adding the absent equivalence, the inferences betray their mediated character:  $(p \rightarrow q) \equiv (\sim p \leftarrow \sim q)$ ;  $(p \rightarrow q) \vdash (\sim p \leftarrow \sim q)$ ;  $(p \vee q) \equiv (\sim p \uparrow \sim q)$ ;  $(p \vee q) \vdash (\sim p \uparrow \sim q)$ , etc.

## IMMEDIATE INFERENCE THROUGH OPPOSITION

In this class are included inferences based on the five relations in the

oppositional hexagon of Robert Blanché (1973).

Knowing the logical relation between two sentences and the truth value of one of them, one can deduce the truth value of the other sentence. If the sentences  $o$  and  $u$  are in a subcontrariety relation, and one of them is false, the other one will be true:  $\{[(o \vee u) \ \& \ \sim o] \rightarrow u\}$ .



1. The oppositional hexagon of Robert Blanché

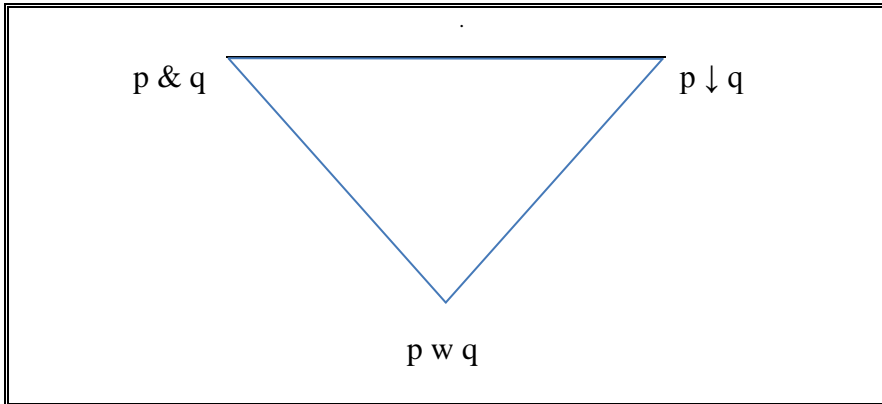
As it can be seen, this approach is no different from a deductive mediated inference. In reality, the structure is simplified, the first premise being tacitly taken, which is why the inference appears as being *immediate*:  $\sim o \rightarrow u$ ; based on the five types of relations existing in the oppositional hexagon, just as many types of simplified inferences were constructed.

*Immediate inferences through contrariety* are those constructed on the basis of the relation between the propositions of the type  $a, e$  and  $y$ , or of the composite sentences,  $p \ \& \ q$ ;  $p \ \nrightarrow \ q$ ;  $p \ \leftarrow \ q$ ;  $p \ \downarrow \ q$  etc., located in the triangle of contrariety.

On the contrariety relation between two sentences and the truth of one of them, the falsity of the other component of the relationship is based, as a conclusion:  $\{(a \ \uparrow \ y) \ \& \ y\} \rightarrow \sim a$ ;  $\{(y \ \uparrow \ e) \ \& \ e\} \rightarrow \sim y$ , etc. The relationship being symmetrical, it allows six inferences of this type. By simplification, those appear as immediate:  $y \rightarrow \sim a$ ;  $e \rightarrow \sim y$ ;  $e \rightarrow \sim a$ ;  $R \ a \ M \rightarrow \sim(R \ y \ M)$ ;  $R \ y \ M \rightarrow \sim(R \ e \ M)$ , etc.

Inferences with composite sentences constructed based on the oppositional hexagonal relations have the same structure as Aristotelian propositions. To satisfy the hexagon relations, composite sentences can occupy certain places.

The sentences  $p \& q$ ;  $p \Rightarrow q$ ;  $p \Leftarrow q$  and  $p \downarrow q$  will be placed on the extreme contraries, on the horizontal line of the triangle, left, right; the middle will be sitting at the bottom tip of the triangle and will be deduced using the truth table, being one of the sentences:  $p \equiv q$ ;  $p \vee q$ ;  $p > q$ ;  $p / q$ ;  $p < q$ ;  $p \setminus q$ .



2. The logical triangle of contrariety<sup>1</sup>, with compound sentences

For example, if the contrary extremes are  $p \& q$  and  $p \downarrow q$ , the mean resulting from the calculation is  $p \vee q$ . Therefore, we have a first triangle with composite sentences, based upon which inferences like the ones discussed can be made:  $(p \& q) \rightarrow \sim(p \downarrow q)$ ;  $p \& q \rightarrow \sim(p \vee q)$ ;  $(p \vee q) \rightarrow \sim(p \& q)$ , etc. The mechanism is the same as explained above: the first premise is missing but is none the less used:  $(p \& q) \uparrow (p \downarrow q)$ ;  $(p \& q) \uparrow \sim(p \downarrow q)$ ;  $(p \& q) \uparrow (p \vee q)$ ;  $p \& q \vdash \sim(p \downarrow q)$ ;  $p \& q \vdash \sim(p \vee q)$ , etc.

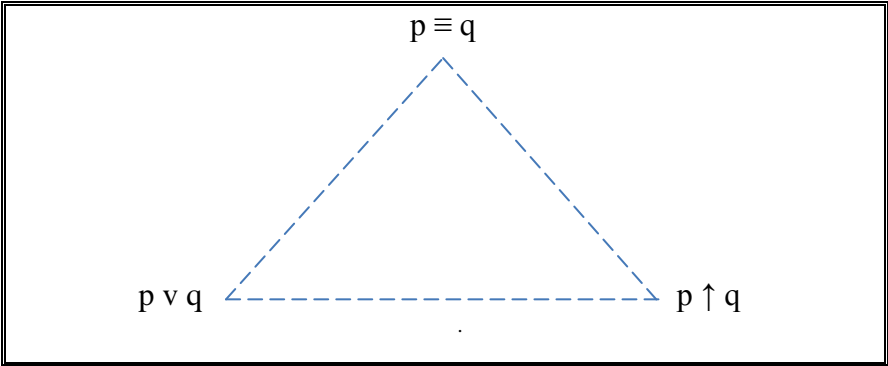
In a symmetrical manner, on the horizontal line of the subcontrary triangle, left, right, the contradictories of the contrary extremes, namely:  $p \uparrow q$ ;  $p \rightarrow q$ ;  $p \leftarrow q$  and  $p \vee q$ , and the middle, the top tip, will be obtained, either by calculation or as a negation of the contrary mean.

*Immediate inferences through subcontrariety* are those inferences constructed based on the relation between the *u*, *i* and *o* type sentences, or the composite sentences,  $p \uparrow q$ ;  $p \rightarrow q$ ;  $p \leftarrow q$  and  $p \vee q$  etc., located in the subcontrariety triangle.

On the relation of subcontrariety between two sentences and the falsity of one of them, the truth of the other is based, as a conclusion:

<sup>1</sup> See Petru Ioan. 1999. *Logica integrală [Integral Logic]*. Volume I. Iași: „Ștefan Lupașcu” Publishing House, p.183.

{[(R o M v R i M) & ¬(R i M)] → R o M}; {[(R u M v R i M) & ¬(R u M)] → R i M} etc. By simplification, they appear as immediate inferences: ¬(R i M) → R o M; ¬(R u M) → R i M; ¬R o M → R u M etc. The situation is similar when using composite sentences. Immediate inferences, (p v q) → (p ≡ q); ¬(p v q) → (p ↑ q); ¬(p ← q) → (p → q) etc., indicates the omission of the premise which specifies the logical relation between the composite sentences involved: (p ≡ q) v (p ≡ q); (p v q) v (p ↑ q), etc.



3. The logical triangle of subcontrariety <sup>2</sup>, with compound sentences

*Immediate inferences through contradiction* are those constructed based on the relations between sentences in total opposition, taken two by two: e, i; a, o; u, y; p ↑ q, p & q; p ≡ q, p w q etc. Being a symmetrical relation, the contradiction allows 12 inferences, using the three lines of the hexagon: M a S → ¬(M o S); ¬(M a S) → M o S; ¬(M o S) → M a S; (M o S) → ¬M a S etc., (p & q) → ¬(p ↑ q); ¬(p & q) → (p ↑ q); (p ↓ q) → ¬(p v q); ¬(p ≡ q) → (p w q), etc.

It is missing from these inferences the justifying premise, which indicates the relation of contradiction between the questioned propositions: (M a S) w (M o S); (p & q) w (p ↑ q); (p ↓ q) w (p v q); (p ≡ q) w (p w q), the reason why these structures are considered immediate.

*Immediate inferences through subalternation* are those constructed based on the logical relation of conditioning expressed by implication. It is an unsymmetrical relation, the hexagon indicating six possibilities: from each sentence in the triangle of contraries, the relationship is reflected in the neighboring sentences. Simplified inference of the

<sup>2</sup>*Ibidem*, p.187.



approach are as follows:  $R a M \rightarrow R i M$ ;  $R a M \rightarrow R u M$ ;  $R e M \rightarrow R o M$ ;  $R e M \rightarrow R u M$  etc., or  $(p \& q) \rightarrow (p \vee q)$ ;  $(p \& q) \rightarrow (p \equiv q)$ ;  $(p \downarrow q) \rightarrow (p \uparrow q)$ ;  $(p \downarrow q) \rightarrow (p \equiv q)$ , etc.

*Immediate inferences through supraalternation* are those constructed based on the conditioning expressed by replication. The relationship is unsymmetrical and takes place in six directions, two from each sentence located in the triangle of subcontrariety. Simplified inferences, which justify the truth value of a sentence from the logical hexagon, have the form:  $\neg(R i M) \rightarrow \neg(R a M)$ ;  $\neg(R i M) \rightarrow \neg(R y M)$ ;  $\neg(R u M) \rightarrow \neg(R a M)$ ;  $\neg(R u M) \rightarrow \neg(R e M)$  etc. or  $\neg(p \vee q) \rightarrow \neg(p \& q)$ ;  $\neg(p \vee q) \rightarrow \neg(p \wedge q)$ ;  $\neg(p \uparrow q) \rightarrow \neg(p \downarrow q)$ , etc.

From the things illustrated along the way, it follows that, through immediate inferences, the truth value of a sentence is justified, knowing the truth value of another sentence with which it is in a determined logical relation. These inferences are mediated structures, which are incompletely expressed: the premise that justifies the relation between the truth-values of the sentences involved in the approach is missing.

The real structure of immediate inferences includes: 1) a premise that expresses the logical relation between two sentences; 2) a premise that specifies the truth value of one of the variables from the first sentence; 3) the conclusion that specifies the truth value of the other variable from the first premise. Therefore, the truth value of the conclusion is based on the conjunction between the logical relation of two sentences and the truth value of one of them. The absent premise provides the rule of conjugation of truth values. It is tacitly assumed, as a well-known principle.

Both inferences through equivalence and through opposition, especially those with composite sentences, require a good understanding of the rules of usage, both for coherent and consistent argumentation, as for combating and rejecting the unjustified ideas of the interlocutor.

Logic remains a science and an art of thinking in which education is essential, and personal effort is mandatory. We consider that logic can't miss in the training of a professional in general, and especially of the one working in education – above all, for the education of thinking.

## REFERENCES:

- Blanché, Robert. 1973. *Le raisonnement*. Paris: P.U. F.  
 Ioan, Petru. 1999. *Logica integrală [Integral Logic]*. Volume I. Iași: „Ștefan Lupășcu” Publishing House.

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- Toma, Melentina. 2004. *Erorile de argumentare, în perspectiva unei tipologii semiotice* [*The Errors of Argumentation in a Semiotic Typology Perspective*]. Iași: „Ștefan Lupașcu” Publishing House.
- Toma, Melentina. 2017. *Raționamentul logic. Raționamentul analitic și înțelegerea unui text scris* [*Logical Reasoning: Analytical Reasoning and Understanding of a Written Text*]. Bucharest: Universul Juridic Press.