THE INFORMAL SIDE OF MATHEMATICS

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Abstract: For the efficiency of mathematics education, one should constantly teach mathematics from its both sides perspective: the (formal) scientific side and the (informal) human, spiritual and cultural side.

Keywords: mathematics, education, philosophy, quality of life, teaching strategies, learning, informal, applications.

Motto: "*God used beautiful mathematics in creating the world*"
Paul Dirac (1902-1984)

INTRODUCTION

At all levels of education, mathematics must be considered as a whole, in its both sides that overlap and reinforce each other: the scientific (formal) side – that includes the whole abstract mechanism (theorems, mathematical formulas, algorithms etc.) and the human and spiritual cultural (informal) side – which refers to the historical and philosophical aspects, to the model function, to knowledge and understanding of different real phenomena, to the important role in scientific inventions, as well as in life quality improvement. Accordingly, the benefits of mathematics can be divided into two profiles: the psycho-motor level and the affective knowledge level (Branzei D. and Branzei R. 2000, Balan and Boncu 1998).

At a psycho-motor level, mathematics develops and orders thinking, making it more disciplined; it forms and develops logical thinking skills, accuracy in expressing ideas, analysis and synthesis abilities; it helps acquire the rigour of logical demonstrations and the technique of logical reasoning; it helps acquire skills for problem-solving, asking questions, finding the best original solutions; it increases attention, concentration and memory capacity; it arouses scientific curiosity, it feeds research interest and it lets in new ideas; it enhances sensitivity to the beauty of fine arts forms, music or

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architecture; it contributes to the development of imagination, creativity, leadership and organizational skills.

From a cognitive-affective level, mathematics helps in understanding our world, modelling real phenomena of physics, chemistry, biology, economics, astronomy, construction, strength of materials, psychology, social sciences, information technology, having an important role in major scientific discoveries and increasing life quality; it offers solutions to real problems, it teaches us how to create and solve problems, it develops intellectual abilities involving various aspects of mathematical understanding, customizing their application in different situations, as well as their use in other contexts; it increases the capacity to train oneself and to overcome one's limits; it enhances some personality traits, based on values and ethics (responsibility towards oneself and towards the others, dignity, sense of initiative, the cult of work, the conscience of work well done as well as the satisfaction of being successful, the propensity for scientific research and better assessment or self-assessment skills).

To generalize one or the other of the two sides is equally wrong and damaging to students. Teaching mathematics only within its cultural, human and spiritual component would mean denying its prevalent formative role of the technical, psycho-motor level, which would be a serious mistake. On the other hand, given the context in which we live nowadays (the rapid and diverse evolution of the information technology, the increased tendency for applied research), one cannot teach only the scientific component of abstract mathematics, as it was the case during the communist regime. Unfortunately, most math teachers do so, and the negative consequences can easily be noticed: pupils/students lose their interest and get bored, they learn less and without any interest, they encounter more difficulties, and they find mathematics difficult, abstract, inaccessible and, even worse, useless.

The informal component of mathematics must be revealed within the education process. It intensifies, motivates and materializes the formal scientific side in a natural and happy way, making students more interested in learning and consolidating mathematics as it enriches their culture and perspectives on the world and life.

In fact, the two components should not be taught successively, isolated from one another, but they must be handled together in a natural interdependence, supporting and building up each other.

HISTORY

The history of mathematics (biographies, legends, stories) softens its abstract and unappealing mechanism, making it more enjoyable and easier to understand and apply.

Mathematics is a wonderful and perfect universe, and many genius creators or common people whose names were lost in the mists of history have contributed to its development since ancient times. For example, it's worth mentioning the name of Hippasos of Metapont, a Pythagorean school philosopher of the 5th century BC, whose legend says he was killed (drowned in accordance with the divine punishment) for the daring offense to the gods of having made public the existence of irrational numbers, which were held secretly as a great danger by the Pythagoreans, whose philosophy was based on the natural numbers.

One might as well mention Hypatia (Alexandria, 370-415 AD), the first woman mathematician in history, also a philosopher and an orator, who is said to have discovered the astrolabe and the hydrometer as well as the first formula of decomposing the cone into flat geometric shapes. She taught astronomy, mathematics, astrology and rhetoric, her intelligence and talent being recognized by the most important Roman universities, at a time when women were excluded from social life. Theon, her father, also a philosopher and a geometer, told Hypatia: “Defend your right to think because, even when thinking wrong, it's better than not thinking at all”. Her life ended tragically, in a period of religious turmoil, when she was killed by a group of fanatic monks, for the courage to have defied the imposed Christianity of that period and to have encouraged the freedom of thinking and choosing a religion. She died for her courage to say: “All formal dogmatic religions are misleading and should never be accepted by people who have self-esteem as absolute beings"; “teaching superstitions that are ranked as truth is really the worst thing in the world "(King 2006)”.

Finally, there is a surprising or rather a normal fact that is worth being mentioned about Mihai Eminescu’s remarkable personality: the great Romanian poet was interested in mathematics, physics or statistics (Săvoiu and Sandulescu). Mihai Eminescu described the main arithmetic operations, such as the addition, the subtraction, the multiplication, the division, the operation of exponentiation and root extraction. Eminescu reviewed mathematical works, too, such as I.P. Eliad's work (*Elements of Arithmetic*, Bucharest, 1876) or D. Petrescu's work (*Elements of Arithmetic*, Bucharest, 1878).

At the end of the 19th century, the biological recapitulation law was discovered: ontogenesis recapitulates filogenesis. Since the very beginning of the 20th century, the education process has been influenced by this famous law which, if transposed into the psychological development, stated that, during their 18 years of schooling, students more or less experience the same stages of humanity development. In other words, along with the learning process, students are experiencing on a small scale the same course of development that science has had in all the history of mankind, since the appearance of man on Earth (Furinghetti and Radford 2002).

When children start going to kindergarten, they learn basic skills, just the same as the first people started learning to communicate and live in a community. Once with the first grade and primary school, children are like the first people who learned how to write and count. Later, while learning geometry, children cross the major Greek discoveries. Once with algebra, pupils acknowledge the flourishing period of the Arabic culture between the 10th and the 13th centuries. Finally, during the years of high school, they study the concepts of the derivative (17th - 18th centuries) and the integrals (18th – 19th centuries). In 18 years, human beings concentrate millions of years of the history of mankind.

PHILOSOPHY

The nature of mathematics, its origins, rationale and some of its fundamental theories were, even in older times, the object of philosophical meditation.

The question of the nature of mathematics refers to two aspects: its objectivity and its certainty. Mathematics fundamentals examine the basic principles and concepts of mathematics, different axioms, concepts or mathematical theories, such as: number, set, category, construct etc. For example, the axiomatic theory of sets analyses the infinite sets (axiom of infinity) or issue admissible mathematical objects (Turlea 2002). In this latter issue, there are more philosophical attitudes: nominalism (according to which there are no abstract or non-spatial entities), conceptualism (which believes that there are also such abstract entities, but only through our mental activity, as mathematics is concerned with studying the topic of non-spatial, yet real, eternal objects), realism (which claims that the laws of mathematics are literal descriptions of the objects of some kind), Platonism (a kind of realism which claims that mathematical objects exist as any other objects).

One should note the realism developed by Gödel; he states that sets and concepts are not located in space and time, but they are independent of us, yet described and, thus, familiar to us). The studies made on some theories and mathematical reasoning (such as Zermelo-Fraenkel's axiomatic study, Gödel's study on the axiom of choice and the continuum hypothesis or Russell's sets paradox) led to the development of the principles and rules governing the functioning of the human spirit and human knowledge - theoretical and practical aspects as well as its limitations.

MEANS OF MODELLING AND WORLD KNOWLEDGE

Mathematics helps us to knowledge and to understand our world, modelling real phenomena in physics, chemistry, biology, economics, astronomy, construction, strength of materials, psychology, social sciences, medicine, information technology, having an important role in the major scientific discoveries.

A simple model is, for example, the formula for calculating interest on the amount borrowed from a bank or for a road map which represents the roads in a particular area.

Mathematics provides models for the influence of the real factors which either imminently or potentially influence climate, for a range of business processes such as, for example, the theory of equilibrium (Arrow and Debreu 1954, Aumann 1965) or exchange (Vind 1964), for social sciences (White 1997, Liginlal and Ow 2006) or for the study of risk factors in predicting bones fractures (Pham, Brandl, N.D. Nguyen and T.V. Nguyen 2008).

It is also worth mentioning the importance of the theory of games and the theory of codes in the automation of calculations, which led to the first calculating machines and the high-performance computers today.

Finally, it is important to emphasize that the logarithm was introduced in the 16th century in order to simplify the complicated calculations of astronomy, that partial differential equations have played an important role in the aviation industry, and that matrix theory was important for the mobile telephony. Without mathematics, we could not have flown a plane and computers or mobile phones would not have existed. The importance and practical usefulness of certain mathematical concepts and theories for technological and cultural development must always be emphasised. Sometimes a small step in mathematics was a huge step for the evolution of mankind.

THE PLAYED ROLE IN ENHANCING THE QUALITY OF LIFE

Since ancient times, people have used mathematical elements in order to enhance their own comfort, daily activities and preoccupations, but also for quality of life.

Long before the appearance of writing, people felt the need to count, probably when, in the course of mankind civilization, "occupations" such as “shepherd” and “farmer” appeared, and man wanted to know how many animals or food supplies he had, what the best timing for farm work was according to the succession of seasons, and how to observe cosmic phenomena (Papuc 2003). According to ancient myths and primitive peoples, it seems that the first figure that appeared was not 1 one, but 2 - it probably came with the first social division of labour between men and women and expressed a pair (two eyes, two hands etc.) or different dual elements (black - white, spirit - matter). The earliest written documents were discovered about 5000 years ago (dry clay tablets, which functioned as real dictionaries), and include knowledge and classifications about animals, plants, elements of medicine, astronomy and geometrical calculations.

Mathematics has developed with people's need to measure land or vessel capacity, to compute time, to solve practical problems in construction, mechanics or navigation.

For example, Archimedes (ca. 287-212 BC) built a mechanical model of the planetary motion system, created mechanisms for lifting some very heavy loads used by the Romans in their battles to defend Syracuse, and invented a system of mirrors that helped the Roman fleet be burned.

Between 1642 and 1645, Pascal (1623-1662) created a machine that performed only two types of operations: addition and subtraction (considered to be the first mechanical computer), out of his desire to help his father who was a tax collector in accounting operations.

Fourier series (which appear in the definition of sine and cosine functions) are used in communication, in signal theory. Thus, the Fourier coefficients are used in designing some filters that allow eliminating the noise (disturbing signals) carried by the communication signals.

In 1944, the theory of games became a useful tool in military strategy within the theory of coding (cryptography). Elements of mathematical logic were used for deciphering coded German messages. The problem was to find a device - mechanism and an automation design, algorithmic mathematics, so that the machine could perform a large number of operations in a sufficiently small period of time. Alan Turing (1912 - 1954) – considered to be the founder of the modern computer – was the one who contributed substantially to the creation of such a mechanism (Turing machine).

The aesthetic importance of mathematics must also be mentioned, as well as its relation to art, architecture and urban planning (Ascher 2005, Castells 1996, Sennott 2004, Rahim 2002). Here a few examples:

- applications of outstanding architectural curves, such as the strofoide, the cisoide, Descartes's foil, the conchoids of Nicomede, the cardioid, Cassini ovals, Bernoulli's lemniscate, the spiral, the cycloid, the necklace;
- the relationship between graphs, urbanism and urban planning;
- the importance of proportion theory in art and architecture (Gheorghiu 2001, Hofsetter 2003, 2005, Radian 1981).

Finally, the connection between music and mathematics must also be mentioned. Using initially a single string instrument (the single-string) and then one with two strings, Pythagoras and his followers found that the sequence of musical notes occurred in constant reports of integers. The Pythagorean musical scale (diatonic Greek range), which was based on three bands (octave 2/1, quint 3/2, block 4/3) was used for writing church music. Later, the Pythagorean scale was modified, resulting in more harmonious tones. Thus, using partial differential equations, Goncharov created new sounds that can lead to the construction of new musical instruments (Dorca 2007, Porphyrios 1998).

TEACHING STRATEGIES

The interdisciplinary and the trans-disciplinary approaches are needed in teaching, as they put a simultaneous emphasis on multiple aspects of children development: the intellectual aspect, the emotional aspect, the social aspect, the physical aspect and the aesthetic aspect. Using interdisciplinary and trans-disciplinary methods, teachers can enrich the information content to be taught and to make themselves to be better understood.

The horizontal transfer of knowledge from one discipline to another is favoured through interdisciplinarity, as it involves the development of connections between certain school subjects so that a concept can be perceived in a uniform and coherent way. An interdisciplinary education can help learners to gain an overview of life and of the universe, to assimilate more thoroughly the fundamental values of mankind (Gavrilut 2010).

Interdisciplinarity creates the correlations among knowledge from different disciplines, the transfer of various methods from one discipline to another, the construction of dynamic and flexible mental structures, as well as the integrated solution to practical problems in personal or social life. Interdisciplinarity and trans-disciplinarity require not only elaborating strategies but, above all, an inner change of attitude and approach of those who educate people.

Here are some directions in this regard:

- to use auxiliary handbooks/teaching aids;
- teachers should reveal the magic and playful side of mathematics, showing games and entertainment issues, interesting problems or some surprising results, paradoxes and mathematical curiosities (by classical means or by using various programs on computers). By doing so, mathematics becomes more fun, more attractive, more easily to address by the new generation of students who are less eager to study or do some research;
- any concept or remarkable achievement should be accompanied by motivations, associations, arguments, connections with other disciplines, historical, philosophical and cultural aspects, contributions and applications in real life. For example, the derivative was defined based on two categories of problems: in physics - mechanical representation (where, for Newton, the derivative was the speed of a mobile object) and in geometry - the question of a tangent to a curve in one point (where for Leibniz, the derivative was the slope of the tangent). Derivatives have many applications in physics (electricity intensity, linear mass density, the law of electromagnetic induction, heat equation, the equation of free oscillation of a transverse bar, telegraph equation for the line without

distortion, etc.), in chemistry (the speed of chemical reactions) or in economy (the marginal cost of production).

Pupils and students need to be convinced (by all the methods and means of teaching) of the role mathematics plays in the technological and economic development of society, in the attempts to improve our living conditions, as well as of the importance of mathematics in the history and evolution of mankind. They must become aware that mathematics – though abstract and barren – is part of the real world. Otherwise, they will always ask why they should acquire some knowledge, why they need it so much, what difference it can make in their future professions.

- the use communicative methods of active participation such as the thinking hats, the conceptual map, the cluster method, the "I know/I want to know/I have learned" method, etc.;
- differential treatment of students, team work;
- informal or extra-curricular activities;
- team teaching can be done by teachers with different specialties, both in the classroom and in scientific circles with pupils or students (either a group of subjects like mathematics - physics - chemistry or mathematics - history – philosophy, or the same subject considered horizontally and vertically);
- the curriculum should stipulate additional hours of mathematics for teaching informal elements - they cannot be covered in two to three minutes within a class;
- the teachers must have a thorough general education, and know the informal aspects as well as the various applications of mathematics in other fields (physics, chemistry, economics).

To strengthening the informal side of mathematics in the learning - assessment process, the following measures may be taken into consideration:

- project-based and exercise-based learning;
- cooperative learning;
- modular organization of the curriculum content;
- optimizing the process of acquiring new knowledge through a presentation that is lively, attractive and applied it;
- in calculating the final grade, a certain percentage could be reserved for a portfolio, which, for example, may contain a report on the life and work of a mathematician, a theme of the history/philosophy of mathematics or a famous problem, some

funny games and exercise, an essay on a book of history of the philosophy of mathematics;

- any written or oral examination should contain topics or questions on the informal side of mathematics;
- the university degree paper or dissertation should contain a paragraph devoted to the informal aspects of mathematics.

QUESTIONNAIRE

In 2008, we used informal mathematical elements in one of the lectures delivered to the students in two groups: in group A, these elements were performed sporadically, and in group B – they were on a regular basis (in each of the 14 weeks of a semester). After a year, students were asked to answer three questions:

a. naming a fundamental mathematical concept (specifically indicated) from the course;
b. writing a mathematical statement (theorem, formula, definition) at their own choice;
c. designating an informal aspect of mathematics.

The number of students who answered the three questions is indicated below:

Group A: No. of interviewed students: 16	Group B: No. of interviewed students: 22
a 2 (12 %)	a 5 (22 %)
b 6 (37 %)	b 12 (54 %)
c 11 (68 %)	c 18 (81 %)

Stating that the average grades of the students in Group B is higher than the average grades of the students in Group A, the results of the questionnaire are relevant and any further comment is unnecessary.

CONCLUSIONS

Mathematics is important not only in creating technical skills and psycho-motor skills of clear logical thinking, based on arguments and algorithms, but it can also contribute to the development of cultural, human-spiritual or emotional-cognitive meanings, as well as to a harmonious character (Heraclitus said: "Human character is his fate"):

- it makes children curious, open-minded, eager to learn and know as much as possible, encouraged to ask questions and seek answers, solve problems and find optimal solutions; it makes them understand mechanisms and phenomena, grasp meanings and decipher enigmas;
- it cultivates honesty, reliability, responsibility, sense of duty and justice, the cult of work, the conscience of work well done, the capacity for decision and action while achieving goals, dignity;
- it develops the competitive spirit and the willingness to work hard, to solve problems, to be successful;
- the informal part of mathematics can effectively fight fear - both the original and the one inherited from the communist era, from parents and grandparents; courage can acquired/learned and taught through mathematics.

Making a child know the usefulness of concepts/relationships and understand the mathematical operation of a facility or law which manifests itself as a real phenomenon, is in itself a great victory of the spirit, but also an important contribution to the cultural level and the scientific potential of the new generation.

Through mathematics one can do miracles in training pupils and this requires will, time, patience and the devotion for mathematics and for teaching it. As teachers, we should be aware of the important role of the educator in the learning process - and this is our mission, as trainers of teachers. The importance of the teaching profession and the teacher who models the human material, who has a unique opportunity to train properly the younger generations should be stressed since the first course in pedagogy or methodology. Future educators must be told that being a professor or a teacher is neither a game, nor something ordinary, but a strategic job implying a lot of responsibility and importance for the nation's healthy future.

Educators need to show generosity, sensitivity, openness and to continually adapt according to the new paradigms of the scientific knowledge - the economic and social changes of the past decades, the rapid and complex development of the information technology require it. It is imperative to remember that behind diversity there is unity and that, beyond the apparent differences, there is something common that binds us and transcends everything.

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